Practice Test 1 (Pearson) Key

Practice Examinations
Calculus AB—Exam 1

Part A—No Calculator

Problem	Answer	Key Concept
1.	(A)	Differentiability
2.	(B)	Graph analysis for extrema
3.	(D)	Definite integral
4.	(C)	Continuity/Differentiability
5.	(C)	Definite integral
6.	(B)	Implicit differentiation
7.	(A)	Definite integral
8.	(B)	Slope field
9.	(D)	L'Hospital's Rule
10.	(D)	Instantaneous rate of change
11.	(A)	Relationships of derivatives
12.	(D)	Limit
13.	(B)	Graph analysis for differentiability
14.	(B)	Velocity/Position
15.	(D)	Fundamental Theorem of Calculus
16.	(B)	Derivative with Chain Rule
17.	(C)	Area approximations
18.	(D)	Tangent line
19.	(C)	Points of inflection
20.	(B)	Integration rules
21.	(B)	Exponential growth derivative
22.	(D)	Graph analysis for points of inflection
23.	(C)	Velocity/Acceleration optimization
24.	(A)	Derivative graph
25.	(D)	Area between curves
26.	(A)	Analysis of derivative data
27.	(D)	Average value
28.	(C)	Numerical derivative
29.	(A)	Rate of change
30.	(A)	Graph analysis for Positon/Velocity

Part B—Calculator Allowed

Problem	Answer	Key Concept
31.	(D)	Graph analysis
32.	(A)	Numerical derivative with calculator
33.	(C)	Derivative graph analysis for extrema
34.	(A)	Critical values
35.	(D)	Continuity/Differentiability
36.	(D)	Fundamental Theorem of Calculus
37.	(B)	Limit
38.	(A)	Exponential growth derivative
39.	(C)	Trapezoidal Rule
40.	(C)	Volume by cross section
41.	(A)	Tangent line
42.	(D)	Fundamental Theorem of Calculus
43.	(A)	Velocity/Acceleration
44.	(C)	Mean Value Theorem/Intermediate Value Theorem
45.	(B)	Area under curve

Practice Test 1 (Pearson) Key

Calculus AB—Exam 1: Section II, Part A

1. $f(x) = g(x)$ at $x = 0$, $x = 1.8109294$, and $x = 3.7724666$. Let $A = 1.8109294$ and $B = 3.7724666$

   (a) Area $= \int_0^A [f(x) - g(x)]dx + \int_A^B [g(x) - f(x)]dx = 9.931$

   (b) Volume $= \pi \int_0^A [(f(x))^2 - (g(x))^2]dx = 64.325$

   (c) Volume $= \int_A^B (g(x) - f(x))^2 dx = 31.790$

2. (a) $v(2.4) \approx -1.452 < 0$ and $a(2.4) \approx 0.884 > 0$. Since the velocity and acceleration have opposite signs, the speed is decreasing at $t = 2.4$.

   (b) Average velocity $= \frac{1}{4}\int_0^4 v(t)dt = 1.158$

   (c) Total Distance $= \int_0^4 |v(t)|dt = 7.309$

   (d) The particle changes from moving left to moving right when the velocity changes from negative to positive.

   $x(2.865) = 2.7 + \int_0^{2.865} v(t)dt = 3.967$

3. (a) $h'(3.5) \approx \dfrac{h(4) - h(3)}{4 - 3} = \dfrac{13.5 - 11}{1} = 2.5$ cm/day

   (b) $\frac{1}{6}\int_0^6 h(t)dt$ is the average length of a leaf in centimeters during the first six days of the data collection.

   $\frac{1}{6}\int_0^6 h(t)dt \approx \frac{1}{6}[1(6.8) + 2(11) + 1(13.5) + 2(20.1)] = \frac{82.5}{6} = 13.75$ cm

   (c) Since the graph of $h(t)$ is strictly increasing on $[0, 6]$, the right Riemann sum is an over-estimate, and therefore greater than the true area.

   (d) $\int_0^6 h'(t)dt = h(6) - h(0) = 20.1 - 5.1 = 15$. The leaf grew a total of 15 cm in the first six days.

Practice Test 1 (Pearson) Key

Calculus AB—Exam 1: Section II, Part B

4. (a) $f'(x) = 0$ at $x = -2$, $x = 0$, and $x = 3$. $x = 0$ is the only critical point where f' changes from negative to positive. Therefore, f has a relative minimum at $x = 0$.

   (b) f is concave up when f'' is positive, which is where f' is increasing. f is increasing where f' is positive. Therefore, f is both concave up and increasing on $0 < x < 1.4$ because f' is both increasing and positive on this interval.

   (c) The graph of f has inflection points at $x = -0.8$, where f' changes from decreasing to increasing and at $x = 1.4$, where f' changes from increasing to decreasing.

   (d) $f(x) = 9 + \int_0^x f'(t)dt$

   $f(-2) = 9 + \int_0^{-2} f'(t)dt = 9 + 2 = 11$

   $f(3) = 9 + \int_0^3 f'(t)dt = 9 + 7 = 16$

5. (a) $\left.\dfrac{dy}{dx}\right|_{(1,1)} = \dfrac{3-2}{6-3} = \dfrac{1}{3} \Rightarrow y - 1 = \dfrac{1}{3}(x-1)$

   (b) $6y - 3x = 0 \Rightarrow 2y = x$

   $(2y)^2 + 3y^2 = 1 + 3(2y)y$

   $4y^2 + 3y^2 = 1 + 6y^2$

   $y^2 = 1$

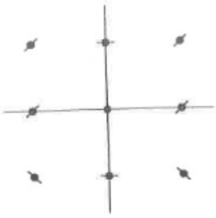
   $y = \pm 1 \Rightarrow x = \pm 2$

   $(-2, -1)$ and $(2, 1)$

   (c) $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{3y-2x}{6y-3x}\right) = \frac{(6y-3x)\left(3\dfrac{dy}{dx}-2\right) - (3y-2x)\left(6\dfrac{dy}{dx}-3\right)}{(6y-3x)^2}$$

   $$\left.\frac{d^2y}{dx^2}\right|_{(1,1)} = \frac{(6-3)\left(3\cdot\frac{1}{3}-2\right) - (3-2)\left(6\cdot\frac{1}{3}-3\right)}{(6-3)^2} = \frac{3(-1)-1(-1)}{9} = -\frac{2}{9}$$

6. (a)



(b) Slopes are positive for points where $x \neq 0$ and $y > -\frac{1}{2}$.

(c) $\int \frac{dy}{2y+1} = \int x^2 dx$

$\frac{1}{2}\int \frac{2dy}{2y+1} = \int x^2 dx$

$\frac{1}{2}\ln|2y+1| = \frac{1}{3}x^3 + C$

$\ln|2y+1| = \frac{2}{3}x^3 + C$

$|2y+1| = e^{\frac{2}{3}x^3+C}$

$|2y+1| = Ce^{\frac{2}{3}x^3}$

$|2(5)+1| = Ce^0$

$11 = C$

$2y + 1 = 11e^{\frac{2}{3}x^3}$

$2y = 11e^{\frac{2}{3}x^3} - 1$

$y = \frac{1}{2}\left(11e^{\frac{2}{3}x^3} - 1\right)$